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[AB]STRACT *(Maximum 200 words)*

A MacCormack time-stepping predictor-corrector method was applied to compressible, subsonic flow about a NACA 0012 airfoil, using a composite van Driest turbulence model and a lowest order boundary condition interpolation. Since good AGARD pressure data was available, it was possible to compare processor times under the two grid systems for identical average pressure (lift) errors. The cartesian system turned out to be less efficient by a factor of only three. Specifically, a 272x145 rectangular grid characterized by dense, point-wasteful bands competed with a 121x45 body oriented grid burdened with coordinate stretching factors. Some elementary analysis accompanying the numerical experiments gives further grounds for believing that cartesian/cubic grids may be only modestly less efficient than the more standard, body oriented ones, while affording far greater generality and requiring far less operator intervention.

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SUITABILITY OF CARTESIAN GRIDS FOR COMPLEX FLOW ANALYSIS

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Introduction

Desirable aspects of a cartesian/cubic coordinate system approach to computational fluid dynamics include its generality, its much reduced claim on the investigator's attention, and its apparent compatibility with CAD (loft) geometry data. Recent noteworthy analyses are presented in Refs. [1] and [2], and in fact nine of the roughly 120 papers presented at that AIAA specialists' conference, Ref. [3], discuss cartesian grids applied to non-square bodies, fields, or flow structures. A literature search shows that cartesian coordinate systems never exited the CFD repertory (though greatly outnumbered by body-fitted systems). In fact, varied problems and speed regimes were so handled through the late 1980s by e.g. American, German, Israeli, Swedish, and Australian investigators in Refs. [4], [5], [6], [7], and [8]. It is conceded that grids aligned with body and flow contours can carry out CFD more efficiently than cartesian grids, but this may become less important in a progressively more bullish computing environment. A relatively unsophisticated

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cartesian CFD routine might be able to handle rather arbitrary body geometries and flow features.

The present study was initiated to enable substantially equal-basis comparisons of these two approaches. (Clearly, unstructured triangular or tetrahedral grids offer yet a third alternative.)

Analysis and Comparison

1. Error Source Contributions

It is presumed that a body-fitted grid not only approximates streamline patterns but is tailored to accurately cover high gradients within boundary layers as well. It is presumed that a cartesian grid achieves general applicability through a crude, successive halving maneuver to cover high gradients wherever they occur, as in Ref. [9].

Since a cartesian system is not aligned with the flow, it has the potential over a stream distance Δs to miss the fluxes within a streamtube of width Δn by the order of $2\Delta y/\Delta n$. But since it relies on close-packing of points, and cells have an aspect ratio $\Delta y/\Delta x=1$, the cartesian system does not miss streamline curvature changes as a streamwise-sparse body-fitted system is like to do. The latter error is characterized by $\Delta\theta=\Delta s/R$, which certainly can be significant; e.g. at a Mach number of 3 a Prandtl-Meyer expansion of just 5 degrees causes static pressure to drop to 2/3 of its initial level. Comparing these, it can be said that the larger-cell, body-aligned coordinate system is not doing importantly better than the smaller-cell, cartesian system unless $\Delta s/R/2\Delta y/\Delta n \ll 1$, i.e. unless $\Delta y \gg \Delta n\Delta s/2R$, which will not be the case if Δy is small enough (fine grid) or if R is small enough (flow turns).

A cartesian system misses body surfaces (just as it misses stream surfaces), and so tends to misapply boundary conditions, unless an interpolation is used. Fortunately, such interpolation schemes are well known, being based upon simple Taylor expansions of the dependent variables about the boundary coordinate. Writing the expansion for successively distant points enables algebraic elimination of the successive derivatives and the establishing of a boundary condition of successively higher order in Δy.

A cartesian system with continued halving more crudely covers the boundary layer regions than a designed system. The latter, ideally, can grid a $u/U=(y/\delta)^{1/7}$ power law with 10 equal increments in u/U by using coordinate spacing $(y/\delta)_i=(i/10)^7$. For a successive halving mechanism to achieve the same 10^{-7} spacing at the bottom of the boundary layer while extending out to $y/\delta=1$, the required number of halvings would be given by $10^{-7}2^N=1$, or $N=23$, which is not excessive compared with 10. So, repeated halving can capture rapidly varying gradients.

2. Numerical Experiments

Comparitive two-dimensional Navier-Stokes calculations were carried out for subsonic, compressible flow about an NACA 0012 airfoil, using MacCormack's time-stepping predictor-corrector method, as described e.g. in Refs. [10] and [11]:

$$\partial Q / \partial t = R \text{ (second spatial derivatives)}$$

A composite van Driest turbulence model as modified and described in Ref. [12] was employed. In the cartesian runs, only a lowest order boundary condition interpolation was used, resulting in an effective steppiness in the airfoil contour and in the resultant static pressure distributions, unless an optional smoothing routine was applied.

The body-fitted grids were generated per the methodology of Ref. [13]. The cartesian grids were designed with high point density bands to ensure sufficient coverage of boundary layers, wakes, and leading and trailing edges. No self-adjustment mechanisms were included in these test cases. Typical grids are shown in Fig. 1.

With essentially the same solver applied to both the body-fitted and cartesian systems, comparisons of machine time for equivalent solution accuracy reflect (1) the inefficiency of cartesian grid point distribution vs. (2) the cost of multiplying coordinate stretching factors in body-fitted grid calculations. The selected residual for convergence tracking was $\partial \varrho u/\partial t$.

Using the grids shown in Fig. 1, flow fields about an NACA 0012 airfoil at zero incidence were calculated. The purpose was to gauge computational efforts to attain comparably accurate results (rather than to strive for maximum accuracy). Measured pressure distribution data was available from Ref. [14], with chosen flow conditions being $M = 0.3$ and $Re = 1.85x10^6$. Equivalent overall error levels were achieved, with the cartesian grid giving better comparison with experiment at forward chord locations and worse comparison rearward, probably indicative of accumulating error in the boundary layer due to the low order of interpolation.

As shown, the ratio of cartesian grid points to body-fitted grid points in these representative calculations was 272x145/121x45=7.24. The corresponding ratio of time steps required for the converged solutions was 3501/1800=1.94, the ratio of CPU per step was 0.929/0.600=1.55, and the ratio of total processing times was therefore 1.94x1.55=3.01. That is, use of the relatively inelegant cartesian system, with its relative profusion of grid points, tripled the claim on computing machine resources.

Further details, and calculations for other cases, are available in Ref. [15].

Conclusion

Substantially equal basis comparisons between body-fitted grid and cartesian grid CFD calculations indicate that the latter require less than an order of magnitude more machine time to achieve equivalent results. Fully general, fully automated cartesian/cubic programs should be developed as one approach to complex-geometry flow problems.

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MISS LI WAS AWARDED THE MASTER OF SCIENCE IN MECHANICAL ENGINEERING BY NC A&T IN MAY 1994.

Figure Captions

Fig. 1 Grid systems utilized in NACA 0012 Navier-Stokes flow field calculations

(a) body-fitted 121x45

(b) cartesian 272x145

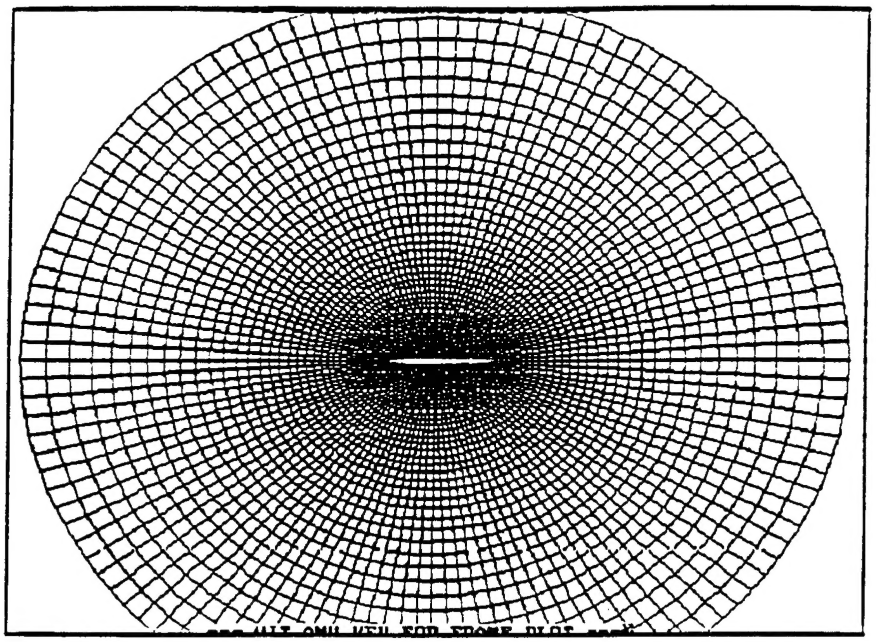

Fig 1(a)

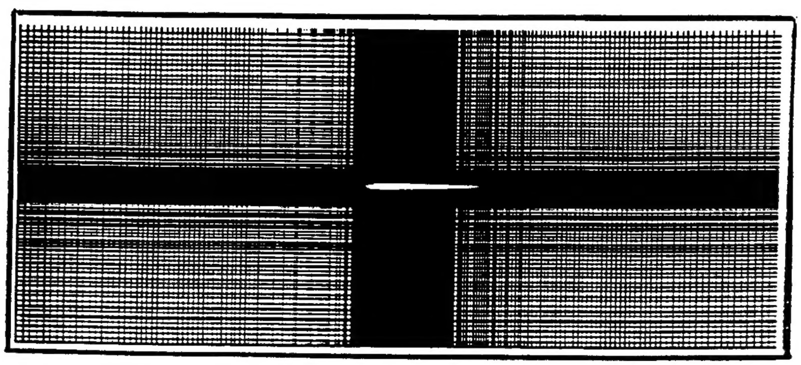

Fig 1(b)